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## **ELMOD - A Model of the European Electricity Market**

Florian Leuthold, Hannes Weigt, and Christian von  
Hirschhausen

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Chair for Energy Economics and  
Public Sector Management

# ELMOD - A Model of the European Electricity Market

Florian Leuthold\*, Hannes Weigt, and Christian von Hirschhausen

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## Abstract

This paper provides a description of ELMOD, a model of the European electricity market including both generation and the physical transmission network (DC Load Flow approach). The model was developed at the Chair of Energy Economics and Public Sector Management ( $EE^2$ ) at Dresden University of Technology in order to analyze various questions on market design, congestion management, and investment decisions, with a focus on Germany and Continental Europe. ELMOD is a bottom-up model combining electrical engineering and economics: its objective function is welfare maximization, subject to line flow, energy balance, and generation constraints. The model provides simulations on an hourly basis, taking into account variable demand, wind input, unit commitment, start-up costs, pump storage, and other details. We report selected study results using ELMOD.

*JEL classifications:* D41; D61; L94

*Keywords:* Electricity markets; Energy pricing; Network modeling

## 1 Introduction

Electricity markets around the world are still in a state of flux, even two decades (for some U.S. markets), one decade (the UK market) or a couple of years (continental Europe) into the reform process. In Europe, the reform

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\*Dresden University of Technology, Faculty of Business and Economics, Chair of Energy Economics and Public Sector Management, 01069 Dresden, Germany. Phone: +49-(0)351-463-39764, Fax: +49-(0)351-463-39763, florian.leuthold@tu-dresden.de

momentum has accelerated in the second half of this decade. In fact, the "Acceleration Directive" (2003/54/EC) has been followed by a more coherent attempt of moving toward a single European market. Yet central reform steps such as vertical unbundling, incentives for cross-border transmission investment, and the integration of large-scale renewable electricity into the network are still in the making. Evidence of this process is provided by the discussions of the "3rd Energy Package" of the European Union, providing energy policy guidelines for the next decade.

In order to understand the impact of different reform proposals and to simulate diverse development scenarios, the Chair of Energy Economics and Public Sector Management ( $EE^2$ ) has developed a model of the European electricity market(s) based on a DC Load Flow model, called ELMOD (Figure 1). The model was initiated by Leuthold et al. (2005) for the German electricity market. Freund et al. (2006) continued this work and extended the model by including France, Benelux, Western Denmark, Austria and Switzerland. Weigt (2006) broadened the scope to a time-frame of 24 hours to simulate variable demand and wind input as well as unit commitment, start-up and pump storage issues. The model was subsequently extended to cover the entire European UCTE electricity markets (essentially Central and Western Europe).

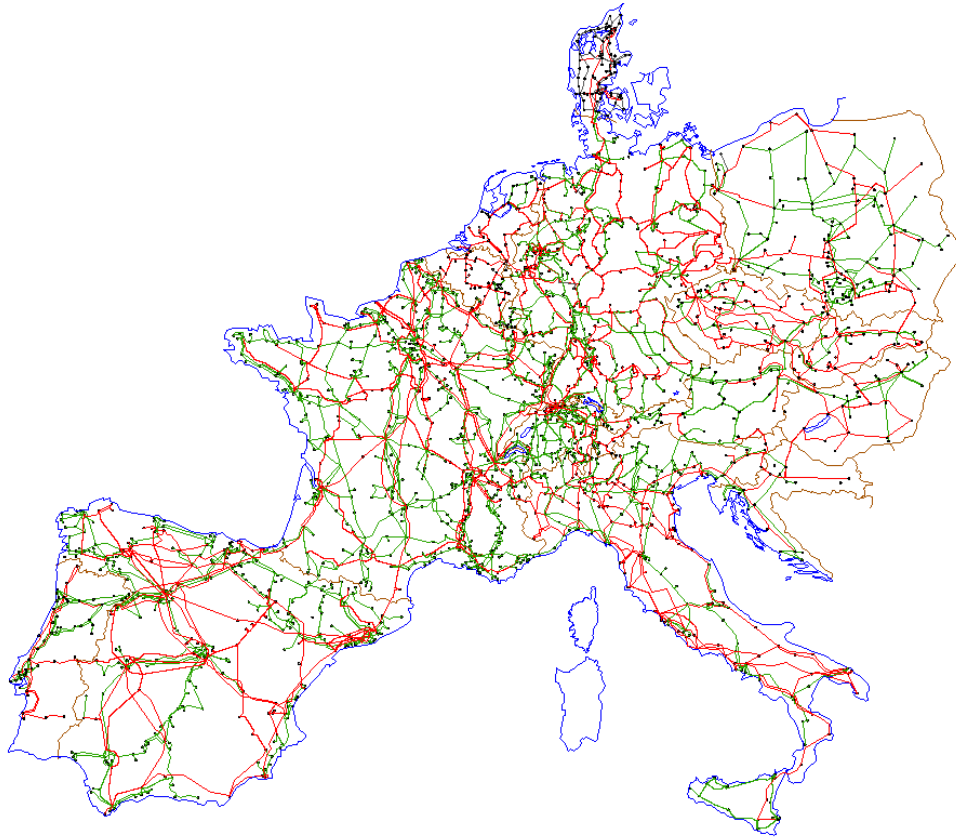
This paper summarizes the current structure of ELMOD and provides an in-depth description of model assumptions and specifics. We start out with an overview of the literature on network modeling (Section 2), and then proceed with the technical and economic details of ELMOD (Section 3). Section 4 presents the data used, the underlying assumptions, sources, et cetera. In Section 5 an overview about previous research results is given including congestion management issues, wind integration, and generation capacity extension. Section 6 concludes and sketches out topics for further research.

## **2 Background and Purpose of the Model**

### **2.1 Survey on modeling electricity markets**

The objective of electricity market reforms is generally to replace monopolistic structures with competition and - where natural monopolies prevail - with more efficient regulation. In Europe, several Directives were issued since 1996 to advance on this reform path. In addition, the discussion of climate change has added further elements to energy policy, such as the European Emissions Trading System (ETS), and the ambitious targets for

**Figure 1:** ELMOD representation of the European high voltage grid



Source: own presentation.

electricity from renewable energy sources, mainly wind. Thus, Germany and Spain have introduced generous feed-in tariffs for onshore and offshore wind energy that the network operators have to integrate in their network management. All in all, there is a strong interest of firms, regulators and scientists in electricity market models taking into account these new challenges of liberalization and changing generation and demand structures.

Ventosa et al. (2005) provide a detailed overview of market modeling tendencies. They point out three trends: optimization models, equilibrium models and simulation models. Optimization models can either apply a profit maximization of a single firm or a welfare maximization approach under perfect competition. Ventosa et al. (2005) distinguish two types of models for a single-firm optimization problem: either the price is an exogenous parameter or determined via a function of the demand supplied by the firm. In contrast, equilibrium models take into account that a firm is able to influence the price by its output decision. The market behavior of all players can then be modeled. Market equilibria problems can either be based on Cournot competition or supply function equilibria differing either in quantity setting or offer curves strategies, respectively. For the time being, equilibrium problems taking into account strategic behavior of many players while considering network constraints are very hard to solve. Ventosa et al. (2005) state that in this case, simulation models can be applied.

Another overview is provided by Smeers (1997) distinguishing between perfect competition models and imperfect competition paradigms. The most simple approach to an ex post analysis of markets seems to use the perfect competition models. Smeers (1997) regards them as very useful since they can handle large data sets and can assess the deviations from perfect markets. Imperfect market characteristics can be introduced into these models as well by taking into consideration quantitative restrictions or mark-ups indicating market power because some agents may be able to charge prices above marginal costs. Furthermore there exists another category of single-staged equilibrium models containing standard imperfect competition paradigms such as the Cournot or Bertrand paradigm and models for system operation. The former being used for ex ante analysis of new institutions like the introduction of a Pool or Power Exchange for electricity. The bases for the latter was introduced by Schweppe et al. (1988), making reference to the concept of economic dispatch: short run operations are assumed to be perfectly regulated, hence its aim is operational. Since electricity cannot be stored, generation and demand have to be equilibrated at any time, making some kind of central control necessary. Smeers (1997) notices that the usual approach to determine generation operations is an economic dispatch model. A third type of models can be found in the multistage equilibrium models being the most complicated and less developed ones. Applications could be investment problems under imperfect competition. There is still a long way to go to make this type of model applicable to large data sets.

In other model reviews such as in Kahn (1998) numerical techniques to analyze market power are examined. In Day et al. (2002) a detailed com-

parison of equilibrium models is accomplished. Classifications are grouped regarding the clearing process used in the power market model (centralized/decentralised) and the nature of interaction among rival generators (from strong competition to collusion). Eight types of equilibrium models are defined including the conjectured supply function. Applications of each model type are indicated. Day et al. (2002) observe that DC load flow approximations are quite common among these models.

Due to the existence of a great variety of market designs both Hogan (2003) and Ma et al. (2003) describe the development towards a standard market design proposed and used in various regions (e.g. already implemented in PJM). Market designs and thus electricity market models drifted in the last decades into two independent directions: on the one hand reliability-driven and on the other hand pricing-driven. After this partial co-existence an optimal Standard Market Design (SMD) was proposed claiming a coordinated spot market for energy and ancillary services. The SMD framework shall include bid-based, security-constrained, economic dispatch implementing locational marginal prices and in particular the introduction of financial transmission rights (Hogan, 2002). Joskow (2005) argues in a similar manner that pure economic models have to be expanded to take the complexity of electrical constraints accurately into account.

## 2.2 Technical specifics and DC Load Flow modeling

Network models have to take into account physical laws when determining prices making electricity an unusual commodity. Electricity cannot be stored, thus requiring demand and supply to equal each other. Furthermore the electricity network transporting electricity from the point of injection to the point of withdrawal has to cope with line capacity limitations, thermal line restrictions, line losses, and security constraints. However, generation and load at any node within the considered network influences the flow on each line, thus demanding quite complex calculations. The use of Kirchhoff's and Ohm's laws is necessary. They include both real and reactive power flows, called AC load flow. An approximation of these load flows for economic modeling can be found in Schweppe et al. (1988), the DC load flow model (DCLF). Schweppe et al. (1988) remark that the name 'DC load flow' is due to historical origins and does not refer to the use of direct current in the electricity network. AC models extend a model's calculation time immensely. Furthermore, AC models may have the problem of non-convergence. In contrast, DCLFs consider only real power equations and can thus reduce the problem size (Overbye et al., 2004). Stigler and Todem

(2005) give a brief but informative insight how to derive the DCLF equations from physical fundamentals. There are two basic assumptions: the voltage angle differences between nodes of the network must be presumed to be very small and the voltage amplitudes to be constant. The main advantage in using a DCLF is its applicability to large scale problems with many capacity constraints and agents (Day et al., 2002).

### 3 Model Description

ELMOD can be classified as a non-linear optimization model maximizing welfare under perfect competition taking into account technical constraints. It is solved in GAMS. ELMOD was originally based on the work of Schweppe et al. (1988) and Stigler and Todem (2005). However, the model underlies a process of developments at  $EE^2$ . Subsequently, first the objective function and the constraints are explained in more detail. Then the DCLF and further modeling specifics such as the representation of demand, of time constraints, and unit commitment are elaborated.

#### 3.1 Optimization problem

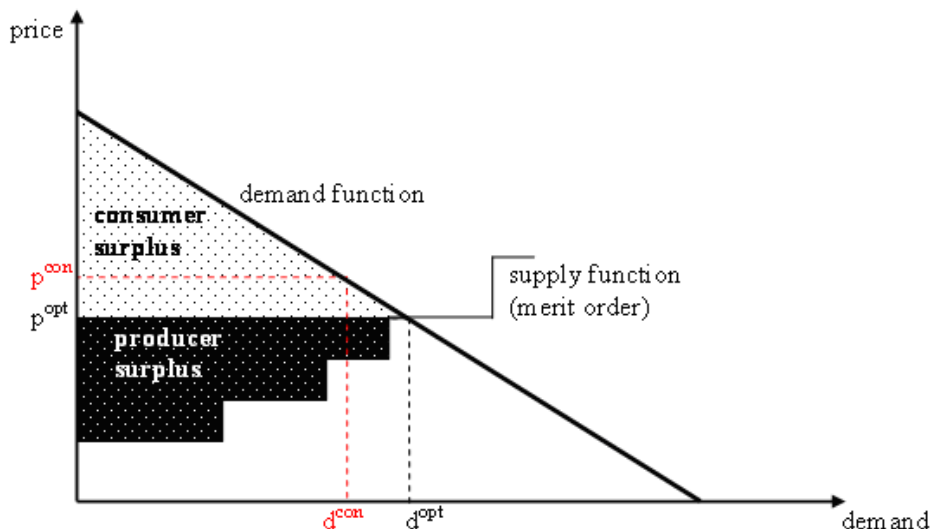
ELMOD uses a welfare maximizing approach taking into account line flow, energy balance and generation constraints. Welfare is obtained using a linear demand and a supply function and can be calculated subtracting the cost of generation from the area below the demand function (Figure 2).

At each node reference demand, reference price and elasticity (see Section 4.3) are estimated in order to identify demand via a linear demand function. Generation cost are determined by an individual cost function for each node. This cost function is composed of a stepwise function joint with a decreasing marginal cost function and cost-blocks for the startup of power plants.

The actual generation costs depend heavily on external parameters such as the fuel price or different efficiency levels of plants which in turn are due to the age or construction of the power plant, the actual level of output and others.

In electricity networks technical constraints have to be considered. Thus a line flow constraint, an energy balance, and a generation constraint are integrated into the model. In the line flow constraint (equation (2)), a maximum amount of power transported  $P_i^t$  on line  $i$  is determined, keeping in mind the thermal limit of each line  $\bar{P}_i$  given a 20% reliability margin. The reliability margin indicates that a line can only be loaded up to 80% of the line capacity thus implementing a simplification of the (N-1)-Criterion.

**Figure 2:** Welfare in an electricity market



Source: own presentation, based on Todem (2004).

The energy balance (equation (3)) at a node  $n$  equals all injections into the grid with all withdrawals corrected by losses. Injections consist of the sum of fossil generation  $\sum_s (g_{ns}^t)$  and wind input  $wi_n^t$ . Pump storage plant generation is added if the pump storage plant generates electricity  $\overleftarrow{PSP}_n^t$ . If the pump storage needs to be filled with water this required electricity  $\overrightarrow{PSP}_n^t$  is subtracted (see also Section 3.4). Generation equals all withdrawals made up of demand  $q_n^t$  and net input  $ni_n^t$  defining whether a node injects or withdraws energy from the grid. The generation constraint in equation (4) assures on the one hand that a power plant  $s$  will be turned off if generation is below a minimum generation  $\underline{g}_{ns}$  necessary to obtain workable technical conditions and on the other hand that it does not exceed its maximum capacity  $\bar{g}_{ns}$ . Each of the constraints must hold for each hour  $t$ . Welfare is derived over all hours<sup>1</sup>:

$$\max_{g_{ns}^t, q_n^t} W = \sum_{n,t} \int_0^{q_n^{t*}} p(q_n^t) dq_n^t - \sum_{n,s,t} (c(g_{ns}^t)g_{ns}^t) \quad (1)$$

<sup>1</sup> A list of the notations can be found in the Appendix.



$$|P_i^t| \leq \bar{P}_i \quad \forall i, t \quad (2)$$

$$\sum_s (g_{ns}^t) + w_n^{it} + \overleftarrow{PSP}_n^t - \overrightarrow{PSP}_n^t - q_n^t - n_n^{it} = 0 \quad \forall n, t \quad (3)$$

$$on_{ns}^t \cdot \underline{g}_{ns} \leq g_{ns}^t \leq on_{ns}^t \cdot \bar{g}_{ns} \quad \forall n, s, t \quad (4)$$

### 3.2 DC Load Flow Model

As stated above, Schweppe et al. (1988) showed that the DCLF can be used for an economic analysis of electricity networks. They apply it to their nodal price approach for electricity pricing. Overbye et al. (2004) come to the conclusion that the DCLF is adequate for modeling nodal prices albeit there are some buses with a certain price deviation. The latter occurs particularly on lines with high reactive power and low real power flows. Stigler and Todem (2005) describe the way from the physical fundamentals to the DCLF equations. Equation (5) of the so-called 'decoupled' AC model builds the foundation of all further assumptions and calculations. Power flow<sup>2</sup>  $P_{jk}^t$  depends on the conductance  $G_{jk}$ , the susceptance  $B_{jk}$ , and the voltage angle difference  $\Theta_{jk}^t$  between nodes  $j$  and  $k$  as well as on the voltage magnitudes  $|U_j|$  and  $|U_k|$ :

$$P_{jk}^t = G_{jk} |U_j|^2 - G_{jk} |U_j| |U_k| \cos \Theta_{jk}^t + B_{jk} |U_j| |U_k| \sin \Theta_{jk}^t \quad (5)$$

Schweppe et al. (1988) assume that the voltage angle difference  $\Theta_{jk}^t$  is very small and that the voltage magnitudes  $|U|$  are standardized to per unit calculations.  $|U_j|$  and  $|U_k|$  are thus assumed to be 1 at each node. Hence the following simplification can be made:

$$\cos \Theta_{jk}^t = 1 \quad (6)$$

$$\sin \Theta_{jk}^t = \Theta_{jk}^t \quad (7)$$

Equation (5) can then be simplified to become:

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<sup>2</sup> The power flow  $P_i^t$  on a line  $i$  can be derived from the power flow  $P_{jk}^t$  between two nodes  $j$  and  $k$  using a network incidence matrix stating which lines  $i$  connect nodes  $j$  and  $k$ . For a more detailed description see Schweppe et al. (1988).

$$P_{jk}^t = B_{jk} \cdot \Theta_{jk}^t \quad (8)$$

Line losses have not been considered yet. However, the sum of total generation does not equal exactly the sum of total demand. Thus, transmission lines are stressed by demand plus losses. In order to approximate the losses on a line, equation (6) must be complemented by the second order term of the Taylor series approximation:

$$\Theta_{jk}^t = 1 - \frac{(\Theta_{jk}^t)^2}{2} \quad (9)$$

Then, after some further assumptions and conversions transmission losses can be calculated via the power flow  $P_{jk}^t$  and the resistance  $R_{jk}$ :

$$P_{Ljk}^t = R_{jk} \cdot (P_{jk}^t)^2 \quad (10)$$

### 3.3 Time constraints, unit commitment, and optimal dispatch

To model electricity markets various idiosyncracies have to be considered. Electricity cannot be stored on a large-scale. Therefore demand and generation always have to equal each other. Demand is not constant over time, but varies in the course of the day, the week and the season. In Europe, demand is higher in winter than in summer mainly influenced by the weather. On workdays more electricity is consumed than on weekends because of a decrease of industrial demand and changed household behavior. To incorporate those characteristics ELMOD models a 24 hours time-frame.

To respond to the varying demand pattern over a day, power plants are divided into three types according to their load type: base load plants supply the grid with a constant output covering thus the base load which is always demanded. Medium load plants provide the increasing electricity demand during the day and are switched on in the morning hours and shut down during the night. Peak load plants are crucial to satisfy various demand peaks during the day. Peak load plants can be turned on within a short time frame.

Unit commitment describes the decision process on whether and when a power plant is running in order to contribute to the satisfaction of demand. Unit commitment identifies those plants available for the following dispatch

process in which the output of each plant is determined exante according to the actual electricity demand, technical needs and the plants cost function. As plants need time to be launched ranging from some minutes for small gas turbines up to several days for large nuclear plants, timing is essential for obtaining a cost minimal dispatch as well as maintaining system stability. ELMOD solves unit commitment within the social welfare optimization process. The optimal output for each plant is determined taking into account the minimal output level to be reached to put a plant online and a certain time for starting up the plant. This introduces a binary variable  $on_{ns}^t$  to the calculation process to determine whether a plant is online or offline. Following Takriti et al. (1998), a minimum online and offline constraint can then be defined:

$$on_{ns}^t - on_{ns}^{t-1} \leq on_{ns}^\tau, \tau = t + 1, \dots, \min\{t + \bar{\vartheta}_s, T\} \quad (11)$$

$$on_{ns}^{t-1} - on_{ns}^t \leq 1 - on_{ns}^\tau, \tau = t + 1, \dots, \min\{t + \underline{\vartheta}_s, T\} \quad (12)$$

Equations (11) and (12) link the hours of the day in order to include online and offline constraints for power plants, respectively. Since the time interval referred to is one hour, only the offline constraint (equation (12)) is used. It is assumed that each plant can be shut down after the end of each hour. Once a plant was shut down, it cannot be turned on again immediately depending on the plant type. Therefore, conditions are introduced to keep plants switched off for a certain time interval  $\underline{\vartheta}_s$ . Further, in order to reduce the calculation effort, each plant is assigned to one group out of three possible groups following Voorspools and D'haeseleer (2003): the must-run units, the peak units and the test group for which the unit commitment process is crucial. Since this is a 24 hour model base load plants such as nuclear and lignite plants are turned on all day long. Hydro plants and gas turbines are supposed to be able to go online within one hour. Hence equation(12) is not binding for them. Thus hard coal plants, oil and gas steam plants, and combined cycle gas turbine plants are within the test group.

Start-up can be distinguished in cold, warm and hot start-up, according to the time since the last shut down. If a plant has recently gone offline, it can be started much faster than a 'cold' plant. This is due to the remaining heat level in the plant, while a 'cold' plant has to entirely build up the necessary starting heat.

The considered time period within the model is one day. Therefore the necessary information to decide on the right kind of start-up may not be available. Also, the calculation effort increases as logic operations have to be

considered. Thus all start-ups for plants within the test group are assumed to be warm ones. For the unconstrained group, all start-ups are supposed to be cold start-ups.<sup>3</sup> The start-up times  $\vartheta_s$  are based on Schröter (2004). Taking these constraints into account, the model calculates the status and the output for each plant in each hour.

### 3.4 Modeling hydro and wind energy plants

Pump storage hydro plants (PSP) as well as wind energy plants cannot be modeled as normal thermal plants. In the case of PSP it has to be considered that energy can either be injected to or withdrawn from the grid. The peculiarity of wind energy is its priority in feed-in. Subsequently, the implementation of these energy types into ELMOD is explained in further detail.

PSPs constitute the only way to store larger amounts of electricity. These plants can run either in pumping mode, filling a storage basin by using electricity, or in generation mode, using the stored water like a classical hydro plant. The electricity therefore is stored in form of potential energy within the water. These plants are crucial for system stability, as they can start-up rapidly and therefore cancel out fluctuations. In general they pump water during night time and weekends and start electricity generation during the peak periods. Within the model, PSPs can either demand the electricity  $\overrightarrow{PSP}_n^t$  and fill their storage or use the stored energy and generate the electricity  $\overleftarrow{PSP}_n^t$ . The pump storage plants are assumed to have an overall degree of efficiency of 75% for pumping and generating, together.<sup>4</sup> The plants start with an empty storage at 8pm. If they run in pump mode, 75% of the consumed energy will be added to the storage. If they run in generation mode the according amount of energy is taken from the storage equation (13):

$$PSP_{storage}^{t+1} = 0.75 \overrightarrow{PSP}_n^t - \overleftarrow{PSP}_n^t + PStore_n^t \quad (13)$$

$$\overrightarrow{PSP}_n^t + \overleftarrow{PSP}_n^t \leq PMax_n \quad (14)$$

$$\overleftarrow{PSP}_n^t \leq PStore_n^t \quad (15)$$

Equations (14) and (15) define the capacity constraints of the storages. The pumped or generated amount is limited by the plant's working capacity

<sup>3</sup> This is irrelevant for the time constraint but important for the cost estimation.

<sup>4</sup> According to Müller (2001), modern PSPs have an average efficiency between 70 and 80%.

$PMax_n$ . The fill level  $PStore_n^t$  of a PSP facility defines the upper bound for the available generation from that facility  $\overline{PSP}_n^t$ .<sup>5</sup>

With 20.6 GW installed capacity at the end of 2006, wind has become a major part of renewable energy produced in the German generation mix (DEWI, 2006). Also on the European level, wind energy is the fastest growing renewable energy source with 48 GW installed in 2006 (EWEA, 2007). Due to the dependence of wind turbines upon wind speed, there is no active control of energy output like in a fossil plant. Only by setting a turbine offline, a minimal active control can be achieved. Because of the feed-in guarantees provided by the Renewable Energy Act in Germany, wind energy has to be injected into the grid and is thus a fixed input for the TSO. Wind speed changes over time according to the meteorological conditions and so does the energy input from wind turbines. In times of high generation by wind turbines, fossil plants must reduce output, while in times of low wind input fossil plants have to compensate the shortfall. A consequence could be additional line flows in the transmission grid, particularly in times of high wind input and low demand.

Wind forecasts play a major role in determining the wind input and therefore the plant schedule for the next hours or day. The differences between forecasted wind input and realized input have to be compensated in order to maintain system stability. The operating reserve that must be provided is not considered in the model. While fossil plants are running in constant mode at an optimal load level whenever possible, wind turbines often run in partial load mode and can change output within hours up to 100%. These changes cause an increased need of backup plants to be able to start-up or reduce output according to the wind input. Within the model, the wind input is calculated for each hour and node and given as an external parameter included in the energy balance (see Section 2).

This constraint can become critical if the grid is not capable of transporting all wind energy. Then the only way to fulfill the energy balance constraint is the increase of local demand even if prices become negative. For the time being, in reality other measures are taken in order to avoid such situations. Possibilities in order to manage such extreme cases are the shut-down of certain wind parks and other technical measures. Such short-term measures are not included in ELMOD.

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<sup>5</sup> Since only one day is simulated, the storage behavior may not be properly modeled, as the storage process largely takes place at weekend nights. Also, the hourly interval may result in a biased representation of PSPs, as one of their main tasks is to react in case of rapidly changing conditions. Since these short time situations are not modeled for the time being, their importance may be underestimated in the model output.

## 4 Data

### 4.1 Grid

The underlying grid is based on the European high voltage grid (UCTE, 2004; VGE, 2006). Substations, line voltage level and line length were uploaded into a digital map, making it possible to add and remove additional lines and nodes. An underestimation of line length can occur, since altitude differences have not been considered. Since no data about the system state is publicly available, all lines connected to a node are assumed to be connected with one another. Also, no information about the transformation capacities of the substations is available. Security constraints are considered by a 20% transmission reliability margin. Thus, no line within the modeled grid will be stressed with more than 80% of their thermal capacity limit.

#### 4.1.1 Germany

The most detailed region mapped in the model is Germany with 365 nodes: 336 regular nodes representing substations and 29 auxiliary nodes. Three different reference line characteristics, one for each voltage level, are assumed within the model, based on Fischer and Kießling (1989). Three main factors are considered: maximum thermal limit, line resistance and line reactance. The values differ significantly for the three voltage levels. To obtain the values for lines with more circuits, the impedances have been calculated according to a parallel combination. Thus, the interaction of multiple circuits has been neglected. The data source for the line characteristics is based on the UCTE-network map (UCTE, 2004). As cross-border flows and transactions play an important role in electricity markets, nine country nodes are added, representing the neighboring countries and 81 cross-border nodes to simulate the import and export, as well as cross-border flows. The model contains 271 lines of the 220 kV and 309 lines of the 380 kV level as well as six lines with 110 kV. In addition, 50 country tie-lines with unlimited capacity are included, connecting the cross-border nodes with the country node and representing the grid of the respective country. Cross-border lines between countries are modeled according to their length and voltage level.<sup>6</sup>

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<sup>6</sup> It must be noticed that the implementation of neighboring countries has an impact on the welfare calculation. As they are part of the overall optimization problem, their demand and generation adds to the total system welfare. Due to energy exports and imports, it is not possible to calculate the welfare for Germany only when including neighboring countries. This must be taken into account while regrading welfare effects. However, as far as only Germany is modeled in detail and the other countries are

### 4.1.2 The European grid

The European UCTE-grid is modeled in a similar way, though with a slightly lower level of detail concerning demand estimations, installed generation capacity, and wind facilities. The entire high voltage grid in Europe is contained in ELMOD based upon the UCTE-network map (UCTE, 2004) as well. The model then covers Portugal, Spain, France, the Netherlands, Belgium, Luxembourg, Western Denmark, Germany, Switzerland, Austria, Italy, Poland, Czech Republic, Slovakia, Hungary and Slovenia. This accounts for about 2120 substations (nodes) and about 3150 lines of the three highest voltage levels. Regarding line characteristics, the same assumptions as for Germany are applied.

## 4.2 Generation

### 4.2.1 Capacities

Generation is divided into eight plant types: nuclear, lignite, coal, oil and gas steam plants, combined cycle gas turbines plants, hydro, pump storage and combined heat power plants. Wind capacity is addressed separately in Section 4.2.3. Power plant capacities are based on VGE (2006). The current database includes all active plants for 2006 with a generation capacity greater than 100 MW. Each plant is assigned to one node. In the case of unclear grid integration, plants are allocated to the geographically closest node. A node can have more than one plant feeding into the grid at this specific node.

Since thermal plants need a certain heat level to produce electricity, a minimal capacity is defined for each plant class according to DENA (2005). These values are specific for every season and identical for every thermal power plant. If output drops below this level, the plant has to be turned off. These values are used for defining the binary plant condition variable indicating if the plant is on- or offline.

Combined heat and power plants (CHPs) often deliver long-distance heat or are integrated in a thermal production process in industries, thus producing electricity as a byproduct. These cogeneration plants were grouped corresponding to their primary output in heat- and power-operated plants. Due to legal guidelines an additional must-run condition was implemented in ELMOD to take into account that energy produced by this type of plant has to be fed-in prior to other energy types. The generation behavior of

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aggregated to a few nodes, the values should largely reflect changes in Germany.

the 'heat-operated' power plants follows the same criteria as other power plants of the same type but they are assumed to be like base load plants in terms of unit commitment. Thus they are always producing at least at their minimum output level which is assumed to be corresponding to the needed heat level.<sup>7</sup> This may lead to an overestimation of output during night times and an underestimation during day times.

#### 4.2.2 Costs

For each plant type a reference efficiency value and marginal cost are estimated based on different fuel types. Depending on the output level a mark-up is added if the output is lower than the reference efficiency value in order to allow for efficiency losses. The mark-ups have been transformed into quadratic polynomials. An additional cost block is added if a thermal plant has to start-up. Hence, cost functions vary between the different plant classes. Also, costs of plants from the same type differ since efficiency levels are not identical. In general, modern plants have a higher efficiency than older ones. However, the construction of the power plant cycle, the actual level of output and external conditions like cooling water availability influence the efficiency as well.

The actual generation costs are calculated on a marginal cost basis. If the output is lower than maximal output, a mark-up is considered to account for efficiency losses. Three mark-ups are defined: one for steam plants, one for combined cycle gas turbine (CCGT) plants and one for gas turbines. The mark-ups depend on the output level in relation to the maximal output. The increase of specific heat consumption due to operating below the optimal output is referred to as partial load conditions (Figure 3). Efficiency can be represented by specific heat consumption.

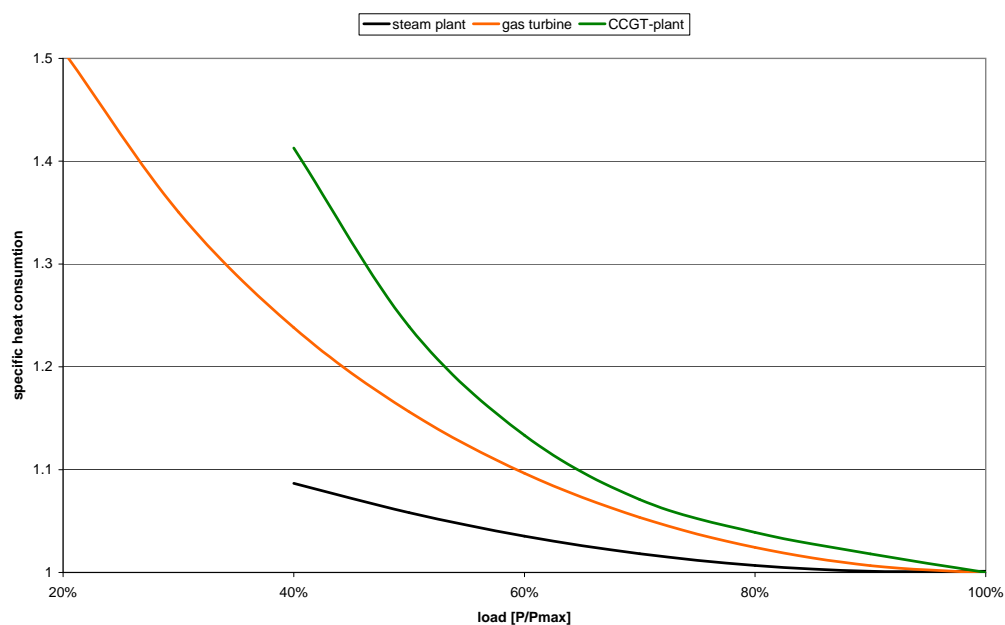
The impact is rather low for classical steam plants, but becomes important for peak load units like gas turbines and therefore is crucial in times of rapidly changing wind input conditions. The mark-up for CCGT-plants is based on VDI (2000) assuming reference efficiency at maximum output of 52.5 % (Müller, 2001). The efficiency of gas and oil fired gas turbines depend on the compressor inlet temperature. Based on a reference efficiency of 34.5% (Müller, 2001) and a temperature level of 15 °C, the partial load efficiency is taken from Kehlhofer et al. (1984). For steam plants, a functional interrelationship of specific heat consumption and partial load can be obtained from Baehr et al. (1985). Nuclear plants may have additional

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<sup>7</sup> Heat demand curves are not included in ELMOD.



**Figure 3:** Partial load efficiency



Source: own calculations

drawbacks due to the necessary security constraints that are not considered within the model formulation.

Based on the above described assumptions it is possible to estimate the impact of varying wind energy on the total system costs. Although wind energy has no marginal generation costs inherently, it causes fossil plants to reduce generation and therefore operate under partial load conditions thus increasing their costs.<sup>8</sup> ELMOD uses the simplified partial load curves in

<sup>8</sup> A simple example reveals the impact: Assume a 1000 MW fossil plant with generation costs of 10 €/MWh that has to reduce its output because 200 MW wind energy are available and need to be fed into the grid. Running at 80% of optimal output causes the efficiency to drop and thereby the costs to rise to 10.07 €/MWh. The cost reduction

order to calculate the cost of wind energy and neglects further wind specific additional costs. Nonetheless the overall impact on welfare is considered. Moreover, prices for  $CO_2$  allowances are included into the generation costs. Therefore the plant specific  $CO_2$  emissions are calculated based on efficiency and plant type according to Gampe (2004). Prices for  $CO_2$  allowances are exogenous to the model and have to be predefined for each study.

Additional costs occur if a thermal plant has to start-up or go offline. Fossil plants generate electrical energy through transforming heat energy. This heat has to reach a certain level before generation can start and has to be cooled down in a controlled process after generation is stopped. The cool-down phase is assumed to be mainly affected by fixed cost parameters. Since ELMOD uses a marginal cost approach, it does not take into account cooling down specifically in its optimization. The start-up costs are mainly driven by fuel prices, as a certain amount of fuel has to be consumed before the heat level is high enough to start electricity generation. The cost estimations for start-up are taken from DENA (2005). These costs are added as a cost block within the hour of start-up. As base load plants are assumed to be must-run plants they do not have start-up costs.<sup>9</sup>

### 4.2.3 Wind

Since wind turbines have relatively small installed capacities, not all of them can be considered individually. To obtain a realistic distribution of wind capacities in Germany a map representing the installed capacity based on  $10km^2$  squares is used (ISET & IWET, 2002). Each square, and therefore a capacity value, is attached to the geographical closest node. This has been done for each federal state separately to obtain a percentage distribution which can then be updated with the actual wind capacities of the federal state. This distribution mechanism also makes it possible to increase the installed capacities without the necessity to reallocate each node individually assuming that installed capacities represent the suitability of a region for the use of wind turbines. As wind input depends on the wind speeds and largely differs between regions, a simplified classification scheme is used. Therefore six different wind zones have been defined using hourly wind speed

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therefore is not 2000 €/h, but only 1944 €/h. The difference could be considered as the indirect marginal cost of wind energy. In reality, a clear cost allocation of wind energy is not possible, because changes in demand modify the operation of the fossil plants. Furthermore, the indirect cost of wind generation is not constant but changes with the load situation of the fossil power plants.

<sup>9</sup> This may lead to biased results in the long run, but should not influence the price and welfare calculation within the modeled reference time frame.

information covering the time from 2002 to 2004 from seven representative stations (DWD, 2005). Since these reference stations are located approximately 10 meter above ground  $H_{ref}$ , an approximation about the speed values in the turbine height is applied: in general, wind speed and height follow a logarithmic function (Hau, 2003):

$$\nu_H = \nu_{ref} \frac{\ln \frac{H}{z_0}}{\ln \frac{H_{ref}}{z_0}} \quad \text{logarithmic height function} \quad (16)$$

Wind speed  $\nu_H$  depends on the absolute height of the turbine  $H$  and the local conditions like the building density, hillsides or forests that influence the roughness length  $z_0$ . To obtain average values a roughness length of 0.2, representing farm land with trees and bushes but without surrounding buildings, is chosen for all nodes. The height of all turbines is assumed to be 60 meters, based on average values for mid-sized turbines. Calculating the speed values for all zones shows a clear separation between the coastal area in the North and the Southern areas.

For wind capacities in Europe, we chose the World Energy Outlook (IEA, 2006) and the Wind Force 12 study (GWEC, 2005). Although both studies analyze the energy economics on a global level and for different time horizons it is possible to extract data for continental Europe. Further data are derived from EMD (2005), EWEA (2005), IG Windkraft (2005), and Wind Service Holland (2005). Wind capacities are allocated according to federal states or similar administrative areas taking into account political, geographical and meteorological framework conditions.

### 4.3 Demand

In order to derive a node-specific demand, ELMOD assumes a positive correlation between economic income and total electricity demand. This relation is modeled in greatest detail for Germany, where demand is differentiated into consumption of industries, services and households: electricity is consumed to around 46% by the industrial sector, 27% by households and 21% by services (EUROSTAT, 2004).<sup>10</sup> Standard load profiles for households (H0) and services (G0) are applied (VDEW, 1999) and are calculated for typical winter and summer workdays. Since various different load profiles

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<sup>10</sup> The remaining electricity consumption is used by agriculture, transport, the energy sector and others. Since these sectors amount only for a small part of the overall consumption, we do not take them into account separately.

exist in the industry sector, we approximate the industry consumption by taking real electricity consumption of a typical winter and summer workday from UCTE (2006) and discount power of households and services according to the standard load profiles. Consequently, the difference indicates the industry consumption. Load profiles are calculated on an hourly basis and are normalized to the overall consumption of electricity made by each sector as stated above.

To weight the sector specific consumption with the amount of this sector on a specific node, we take the gross value added of industry and services and the gross domestic product considering households. The gross value added is available at Euro NUTS 3 level. Each district is assigned to a node. In case there are different nodes in one district, the whole gross value is divided by the number of nodes. In case there is no node in the district, the gross value added is distributed to all neighboring districts with nodes. The share of a node of the whole gross value added is calculated and applied to the overall electricity consumption by industry and services, respectively. Regarding the node-specific consumption of households, they are deduced distributing the inhabitants of an administrative district to the node in the same manner as the gross value added for industry and services are assigned to. In a second step, the annual energy consumption of the households is assigned to the nodes according to the node's share in the whole gross domestic product. This, subsequently, yields a reference demand per node. On the basis of this reference demand, a reference price (e.g. average EEX price for Germany) and the assumption of a demand elasticity at this reference point (e.g. of  $-0.25$ ), a linear demand function can be estimated.

For the remainder of Europe, demand is based on UCTE data. For models with focus on Germany the neighboring countries are condensed in single nodes, thus a separation of demand according to industry, commerce and residential is not necessary. Reference prices are taken from the national electricity exchanges.<sup>11</sup> A linear demand behavior is obtained in the same way as for Germany. For studies covering more countries a node specific demand is derived by using the gross value added as key for a distribution of load to different districts. Thus, a separation of household, service and industrial demand is not considered for the rest of Europe.

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<sup>11</sup> In case no national price is available, a European average price is calculated based on the existing national prices.

## 5 Applications of ELMOD

### 5.1 Network constraints and offshore wind

ELMOD was initially used in order to study different congestion management schemes for the German electricity market, particularly the problem of integrating large scale offshore wind projects as presented in DENA (2005). Leuthold et al. (2005) demonstrate that nodal pricing is superior to uniform pricing and conclude that when using nodal pricing, 8 GW offshore wind capacities can be implemented without grid extension and additional 5 GW if the North West German grid will be extended. As the underlying model is time static, varying demand and wind input are considered through different reference cases. Also, cross-border flows and unit commitment decisions are neglected. Freund et al. (2006) continued the work and extended the model by including France, Benelux, Western Denmark, Austria and Switzerland. Therefore, they could also examine cross-boarder flows. Freund et al. (2006) point out that, even under status quo conditions, the price situation in Benelux is affected by high wind input in Germany. This situation is bound to aggravate if the planned wind capacity extension will be realized without proper grid adjustments. The work of Freund et al. (2006) is the first approach to model the effects of nodal pricing in combination with increased wind energy on the North-Western European grid. Weigt (2006) extended the model by including a time-frame of 24 hours to simulate variable demand and wind input as well as unit commitment, start-up and pump storage issues. He shows that for the German market a nodal pricing system would yield significantly lower prices during peak times on average. The impact of wind energy under current conditions is mainly predictable and leads to price decreases in North and East Germany. However, in specific load and wind input cases congestion situation can lead to price increases in South Germany. The planned wind capacity extensions based on a forecast for 2010 lead to significant price reductions in North Germany but increase price differences particularly between the Netherlands and Germany as well as between South and North Germany. The problem of grid extensions due to increased wind input is taken up by Jeske (2005) and Jeske et al. (2007). Jeske (2005) analyzes the possibility of integrating large scale offshore capacities using high voltage direct current (HVDC) lines in order to transport the energy to demand centers in the South and West of Germany. He finds that when applying welfare criteria and considering congestion, the HVDC-approach is more efficient than other grid extension measures. Jeske et al. (2007) analyze the additional grid investments necessary to integrate

the projected wind capacities forecasts for Europe in 2020. They conclude that the UCTE grid seems to be prepared for large amount of new wind capacities but requires extensions particularly in cross-border capacities.

## 5.2 Locating generation investments

Dietrich et al. (2007) applied ELMOD in order to model optimal investment behavior up to the year 2012 based on realistic data of planned generation investments. They represent an average year in terms of demand and wind levels. Twelve cases are defined to simulate off-peak, mid-load and peak demand in winter and summer as well as high and average wind input. Analyzing locations of plants yielded different results for different grid extension scenarios. While the projected locations were mainly along the North-Sea coastline and the Ruhr area, the optimal model results for locations varied significantly with assumptions regarding the grid situation. To put it in a nutshell, Dietrich et al. (2007) show that transmission expansion is a critical condition for generation investment locations, particularly in a European context.

## 6 Conclusions

In this paper, we have presented the current version of ELMOD, a welfare maximizing engineering and economic model of the European electricity market, developed at the Chair of Energy Economics and Public Sector Management ( $EE^2$ ) at Dresden University of Technology. ELMOD is based on a DC Load Flow approach and captures the essentials of the European electricity markets, even though it lacks some idiosyncrasies of some national markets. ELMOD can be applied to analyze the effect of offshore wind power on the North-West European electricity market, and the effects of congestion between countries and within the German grid. Additionally, ELMOD can also be used applied to generation investment issues namely the siting of new power plants under grid constraints. Further development steps of ELMOD are to endogenize investment decisions, in particular the interdependence between investments in generation and in transmission. On the long run, it might be worth the while to integrate strategic behavior of at least one integrated player, and to introduce stochastic elements into the model.

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## Appendix

### Abbreviations, Nomenclature and Indices

<b>Abbreviations</b>	
AC	Alternating Current
CCGT	Combined cycle gas turbine
CHP	Combined Heat and Power plant
CHP	Combined heat and power plant
DC	Direct Current
DCLF	Direct Current Load Flow
DENA	Deutsche Energie-Agentur (German Energy Agency)
DEWI	Deutsches Windenergie-Institut (German Wind Energy Institute)
EEG	Law on Renewable Energies in Germany
EEX	european energy exchange
ELMOD	Model of the European electricity grid
GAMS	General Algebraic Modeling System
GW	Gigawatt
HVDC	High Voltage Direct Current
kV	Kilovolts
MW	megawatt hour
NUTS	Nomenclature des Unités Territoriales statistiques
PJM	Pennsylvania-New Jersey-Maryland Transmission Organization
PSP	Pump Storage Hydro plants
SMD	Standard Market Design
UC	Unit Commitment
UCTE	Union for the Coordination of Transmission of Electricity
VDEW	Verband der Elektrizitätswirtschaft e. V. (Association of Electricity Economics)
VDI	Verein Deutscher Ingenieure (Association of German Engineers)
<b>Nomenclature</b>	
$\nu$	wind speed [m/s]
$\overleftarrow{PSP}_n^t$	PSP generation [MW]

$\bar{\vartheta}_s$	minimum online time of plant type s	$p_n^t$	price at node n [€/MWh]
$\bar{g}_{ns}$	maximum generation capacity at node n [MW]	$P_{jk}$	real power flow between two nodes [MW]
$\bar{P}_i$	maximum transmission capacity on line i [MW]	$P_{Ljk}$	losses of real power between two nodes [MW]
$\overrightarrow{PSP}_n^t$	PSP upload [MW]	$PMax_n$	maximum generation of pump storage at node n [MW]
$\Theta_{jk}^t$	voltage angle difference [rad]	$PStore_n^t$	storage amount at node n [MW]
$\underline{\vartheta}_s$	minimum offline time of plant type s	$q_n^t$	demand at node n [MWh]
$\underline{g}_{ns}$	minimum generation capacity at node n [MW]	$q_n^{t*}$	equilibrium demand at node n [MWh]
$B_{jk}$	line series susceptance [1/Ω]	$U_j$	voltage magnitude at a node [volts]
$c(g_{ns}^t)$	costs function depending on the level of production [€]	$U_k$	voltage magnitude at a node [volts]
$G_{jk}$	line series conductance [1/Ω]	$W$	welfare [€]
$g_{ns}^t$	generation at node n of plant type s not including wind and PSP [MW]	$w_n^{it}$	total generation of wind energy at node n [MW]
$H$	height [m]	$z_0$	roughness length
$ni_n^t$	net input per node n [MW]	<b>Indices</b>	
$on_{ns}^t$	binary plant condition variable (on = 1, off = 0)	$i$	line between node j and node k
$P_i^t$	real power flow at line i [MW]	$j$	node within the network
		$k$	node within the network
		$n$	nodes within the network

*ref* reference  
*s* plant type

*t* time period